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Theory of friction: the contribution from a fluctuating electromagnetic field

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Abstract. We calculate the friction force between two semi-infinite solids in relative parallel motion (with velocity V), and separated by a vacuum gap of width d. The friction force results from coupling via a fluctuating electromagnetic field, and can be considered as the dissipative part of the van der Waals interaction. We consider the dependence of the friction force on the temperature T, and present a detailed discussion of the limiting cases of small and large V and d.

1. Introduction

Because of its great practical importance and because of the development of new experimental techniques, sliding friction has become a topic attracting increasing attention [1]. In this paper we consider the friction force between two solids in relative motion, separated by a vacuum gap of width d. This 'vacuum' friction is in most cases of no direct practical importance since the main contribution to the friction force when a body is slid on another body comes from the area of real (atomic) contact [1]. Thus, the frictional stress between two semi-infinite metallic (e.g., copper) bodies, moving parallel to each other with the relative velocity $V = 1$ m s^{-1}, and separated by the distance $d = 10$ Å, is only (see reference [2] and below) $\sim 10^{-6}$ N m^{-2}. This stress is extremely small compared with the typical frictional stress, $\sim 10^{8}$ N m^{-2}, occurring in the area of atomic contact even for (boundary) lubricated surfaces. Nevertheless, vacuum friction is important in some special cases (see reference [2]), and determines the ultimate limit to which friction can be reduced. Quantum and thermal fluctuations of the polarization and the magnetization of solids give rise to a fluctuating electromagnetic field. For two stationary solids the interactions mediated by this field result in the well-known attractive van der Waals force. For two solids in relative motion this interaction will also give rise to a friction force between the bodies. The static aspect of the van der Waals interaction is well understood but there are still controversial results concerning the dynamical part. Different authors have recently studied the van der Waals friction using different approaches, and obtained results which are in sharp contradiction to each other. The first calculation of van der Waals friction was done by Teodorovich [3]. Schaich and Harris [4], and Pendry [5] argue that Teodorovich's calculation is in error. For two metallic bodies Schaich and Harris found that the friction force is independent of any metal property, in contrast to the results of other authors. The friction forces calculated by Levitov [6], Polevoi [7] and Mkrtchian [8] vanish in the nonretarded limit (formally obtained when the light velocity $c \to \infty$). This result is very surprising (and in our opinion incorrect), since neglecting retardation is a good approximation at short separations d

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between the surfaces, in which case one would expect the friction force to be particularly large. Even at large separation, where the nonretarded contribution is negligible, our result differs form those of Levitov, Polevoi and Mkrtchian. Pendry [5] considered only the case of zero temperature, and Persson and Zhang [2] the case of small sliding velocities, and both groups neglected retardation effects. To clarify the situation we present a straightforward calculation of the van der Waals friction based on the general theory of the fluctuating field developed by Rytov [9] and applied by Lifshitz [10] for studying the conservative part of the van der Waals interaction. In this approach the interaction between the two bodies is mediated by the fluctuating electromagnetic field which is always present in the vicinity of any collection of atoms. Beyond the boundaries of a solid this field consists partly of travelling waves and partly of evanescent waves which are damped exponentially with the distance away from the surface of the body. The method that we use for calculating the interaction forces is quite general, and is applicable to any body at arbitrary temperature. It also takes into account retardation effects, which become important for large enough separation between the bodies. A similar approach was used by Polevoi [7] but he obtained a nonzero friction force only in the relativistic limit, in contrast to the present calculations and the earlier calculations of Persson and Zhang [2], and Pendry [5]. Polevoi did not give enough details of his calculation to compare his theory with the present calculation, but we believe that he overlooked effects related to the change in the reflectivity of electromagnetic waves from moving bodies, which occur even in the nonrelativistic limit. In the nonretarded limit and for zero temperature the present calculation agree with the results of Pendry. Similarly, in the nonretarded limit and for low sliding velocities, we agree with the study of Persson and Zhang.

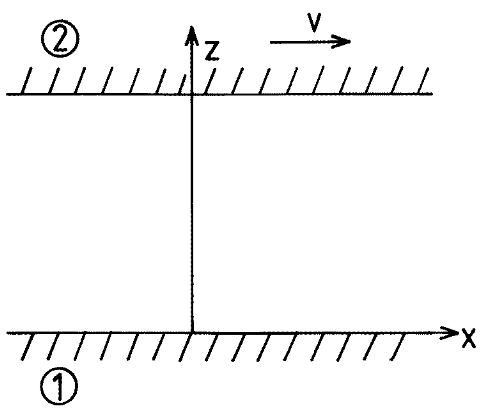

Figure 1. Two semi-infinite solids having flat parallel surfaces separated by a distance d and moving with velocity V relative to each other.

2. Calculation of the fluctuating electromagnetic field

We consider two semi-infinite solids having flat parallel surfaces separated by a distance d and moving with velocity V relative to each other; see figure 1. We introduce the two coordinate systems K and K' with coordinate axes xyz and $x'y'z'$. In the K-system, body **1** is at rest while body **2** is moving with velocity V along the x-axis (the xy- and $x'y'$-planes are in the surface of body **1**, the x- and x'-axes have the same direction, and the z- and z'-axes point toward body **2**). In the K'-system, body **2** is at rest while body **1** is moving with velocity $-V$ along the x-axis. Following Lifshitz to calculate the fluctuating field in the interior of the two bodies, we shall use the general theory which is due to Rytov and is described in detail in his book [9]. This method is based on the introduction into the Maxwell equations of a 'random' field (just as, for example, one introduces a 'random' force in the theory of Brownian motion). In the K-system in a dielectric, nonmagnetic body **1** for a monochromatic field (time factor $e^{-i\omega t}$) these equations are

$$\begin{aligned} \nabla \times \boldsymbol{E}_1 &= i\frac{\omega}{c}\boldsymbol{B}_1 \\ \nabla \times \boldsymbol{B}_1 &= -i\frac{\omega}{c}\varepsilon_1(\omega)\boldsymbol{E}_1 - i\frac{\omega}{c}\boldsymbol{F}_1 \end{aligned} \tag{1}$$

where $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic fields, $\varepsilon_1(\omega)$ is the complex dielectric constant for body **1**, and $\boldsymbol{F}$ is the random field. According to Rytov the correlation function of the latter, determining the average value of the product of components of $\boldsymbol{F}$ at two different points in space, is given by the formula

$$\langle F_i(x, y, z)F_k(x', y', z')\rangle = 4\hbar\left(\frac{1}{2} + n(\omega)\right)\varepsilon''(\omega)\delta_{ik}\delta(x - x')\delta(y - y')\delta(z - z') \tag{2}$$

where

$$n(\omega) = \frac{1}{e^{\hbar\omega/k_B T} - 1}$$

and where T is the temperature and ε'' is the imaginary part of $\varepsilon = \varepsilon' + \varepsilon''$. The function $\boldsymbol{F}(x, y, z)$ can be represented in the form of a Fourier integral, which can be written for the half-space $z < 0$ in the form

$$\boldsymbol{F}(x, y, z) = \int_{-\infty}^{+\infty} \boldsymbol{g}(\boldsymbol{k})e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \cos k_z z \, d^3k \tag{3}$$

where a two-dimensional vector $\boldsymbol{q}$ is parallel to the surface, $k^2 = k_z + q^2$, and $\boldsymbol{r}$ is the radius vector in the xy-plane. For the Fourier components $\boldsymbol{g}(\boldsymbol{k})$, the correlation function corresponding to the spatial correlation (3) is

$$\langle g_i(\boldsymbol{k}, \omega)g_k^*(\boldsymbol{k}', \omega)\rangle = \frac{\hbar(\frac{1}{2} + n(\omega))\varepsilon''(\omega)}{\pi^3}\delta_{ik}\delta(\boldsymbol{k} - \boldsymbol{k}'). \tag{4}$$

For body **1** ($z < 0$) the fields $\boldsymbol{E}$ and $\boldsymbol{B}$ can be written in the form [10]

$$\boldsymbol{E}_1 = \int_{-\infty}^{+\infty} \{\boldsymbol{a}_1(\boldsymbol{k}) \cos k_z z + i\boldsymbol{b}_1(\boldsymbol{k}) \sin k_z z\} \, e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \, d^3k + \int_{-\infty}^{+\infty} \boldsymbol{u}_1(\boldsymbol{q})e^{i\boldsymbol{q}\cdot\boldsymbol{r} - is_1 z} \, d^2q \tag{5}$$

$$\begin{aligned} \boldsymbol{B}_1 = \frac{c}{\omega}\int_{-\infty}^{+\infty} &\{([\boldsymbol{q} \times \boldsymbol{a}_1] + k_z[\boldsymbol{e}_z \times \boldsymbol{b}_1]) \cos k_z z + i([\boldsymbol{q} \times \boldsymbol{b}_1] + k_z[\boldsymbol{e}_z \times \boldsymbol{a}_1]) \sin k_z z\} \, e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \, d^3k \\ &+ \frac{c}{\omega}\int_{-\infty}^{+\infty} \{[\boldsymbol{q} \times \boldsymbol{u}_1] - s_1[\boldsymbol{e}_z \times \boldsymbol{u}_1]\} \, e^{i\boldsymbol{q}\cdot\boldsymbol{r} - is_1 z} \, d^2q \end{aligned} \tag{6}$$

where $\boldsymbol{e}_z$ is a unit vector in the direction of the z-axis, and

$$s_1 = \sqrt{\frac{\omega^2}{c^2}\varepsilon_1 - q^2} \tag{7}$$

where the sign of the root is to be chosen such that the imaginary part of s will be positive.

The first terms in the expressions (5) and (6) represent a solution of the inhomogeneous equations (1). Substituting them in the second equation of (1) and writing $\boldsymbol{F}$ in the form (3), one can find the following relations, expressing $\boldsymbol{a}_1$ and $\boldsymbol{b}_1$ in terms of the Fourier components $\boldsymbol{g}_1$ of the random field:

$$\boldsymbol{a}_1 = \frac{1}{\varepsilon_1(k^2 - \omega^2\varepsilon_1/c^2)}\left[\frac{\omega^2}{c^2}\varepsilon_1\boldsymbol{g}_1 - \boldsymbol{q}(\boldsymbol{q} \cdot \boldsymbol{g}_1) - k_z^2 g_{1z}\boldsymbol{e}_z\right] \tag{8}$$

$$\boldsymbol{b}_1 = -\frac{k_z}{\varepsilon_1(k^2 - \omega^2\varepsilon_1/c^2)}\left[\boldsymbol{e}_z(\boldsymbol{q} \cdot \boldsymbol{g}_1) + \boldsymbol{q}g_{1z}\right]. \tag{9}$$

The second integrals in equations (5), (6) represent the solution of the homogeneous equations (1) (i.e. the equations with $\boldsymbol{F}$ omitted), and describe the plane-wave field reflected from the boundary of body. The condition for transversality of these waves is

$$\boldsymbol{u}_1 \cdot \boldsymbol{q} - s_1 u_{1z} = 0. \tag{10}$$

In the space between bodies (vacuum), $\varepsilon = 1$, $\boldsymbol{F} = \boldsymbol{0}$ and the field in the K-system is given by the general solution of the homogeneous equations, which can be written in the form

$$\boldsymbol{E}_3 = \int_{-\infty}^{+\infty} \left\{ \boldsymbol{v}(\boldsymbol{q}, \omega)\mathrm{e}^{\mathrm{i}pz} + \boldsymbol{w}(\boldsymbol{q}, \omega)\mathrm{e}^{-\mathrm{i}pz} \right\} \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}\, \mathrm{d}^2 q \tag{11}$$

$$\boldsymbol{B}_3 = \frac{c}{\omega} \int_{-\infty}^{+\infty} \left\{ ([\boldsymbol{q} \times \boldsymbol{v}] + \boldsymbol{p}[\boldsymbol{e}_z \times \boldsymbol{v}])\mathrm{e}^{\mathrm{i}pz} + ([\boldsymbol{q} \times \boldsymbol{w}] - \boldsymbol{p}[\boldsymbol{e}_z \times \boldsymbol{w}])\mathrm{e}^{-\mathrm{i}pz} \right\} \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}\, \mathrm{d}^2 q \tag{12}$$

where

$$p = \sqrt{\frac{\omega^2}{c^2} - q^2} \tag{13}$$

and $\boldsymbol{v}$ and $\boldsymbol{w}$ satisfy the transversality conditions

$$\boldsymbol{v} \cdot \boldsymbol{q} + p v_z = 0 \qquad \boldsymbol{w} \cdot \boldsymbol{q} - p w_z = 0. \tag{14}$$

The boundary conditions on the surfaces of the media are the requirement of continuity of the tangential components of $\boldsymbol{E}$ and $\boldsymbol{B}$ in the rest frame of the respective body. In the K-system, on the plane $z = 0$ for a given value of $\boldsymbol{q}$ it is convenient to write the corresponding equations for components of the fields along the vectors $\boldsymbol{e}_{\boldsymbol{q}} = \boldsymbol{q}/q$ and $\boldsymbol{e}_{\boldsymbol{n}} = [\boldsymbol{e}_z \times \boldsymbol{e}_{\boldsymbol{q}}]$; this gives the following equations:

$$\begin{aligned}
&\int_{-\infty}^{+\infty} a_{1\boldsymbol{q}}\, \mathrm{d}k_z + u_{1\boldsymbol{q}} = v_{\boldsymbol{q}} + w_{\boldsymbol{q}} \\
&\int_{-\infty}^{+\infty} a_{1\boldsymbol{n}}\, \mathrm{d}k_z + u_{1\boldsymbol{n}} = v_{\boldsymbol{n}} + w_{\boldsymbol{n}} \\
&\int_{-\infty}^{+\infty} (q a_{1z} - k_z b_{1\boldsymbol{q}})\, \mathrm{d}k_z + q u_{1z} + s_1 u_{1\boldsymbol{q}} = q(v_z + w_z) - p(v_{\boldsymbol{q}} - w_{\boldsymbol{q}}) \\
&\int_{-\infty}^{+\infty} -k_z b_{1\boldsymbol{n}}\, \mathrm{d}k_z + s_1 u_{1\boldsymbol{n}} = -p(v_{\boldsymbol{n}} - w_{\boldsymbol{n}})
\end{aligned} \tag{15}$$

where $a_{1\boldsymbol{q}} = \boldsymbol{e}_{\boldsymbol{q}} \cdot \boldsymbol{a}_1$, $a_{1\boldsymbol{n}} = \boldsymbol{e}_{\boldsymbol{n}} \cdot \boldsymbol{a}_1$ and so on. In what follow we shall need only the field between two media. Using the transversality conditions (10) and (14) and the expressions (8) and (9), from equations (15) we can obtain the following equations:

$$\begin{aligned}
p s_1 &\int_{-\infty}^{\infty} (q g_{1z}(\boldsymbol{q}, k_z, \omega) - s_1 g_{1\boldsymbol{q}}(\boldsymbol{q}, k_z, \omega)) \frac{1}{k_z^2 - s_1^2}\, \mathrm{d}k_z \\
&= -(s_1 + p\varepsilon_1) v_{\boldsymbol{q}}(\boldsymbol{q}, \omega) - (p\varepsilon_1 - s_1) w_{\boldsymbol{q}}(\boldsymbol{q}, \omega)
\end{aligned} \tag{16}$$

$$s_1 \left(\frac{\omega}{c}\right)^2 \int_{-\infty}^{\infty} \frac{g_{1\boldsymbol{n}}(\boldsymbol{q}, k_z, \omega)}{k_z^2 - s_1^2}\, \mathrm{d}k_z = (p + s_1) v_{\boldsymbol{n}}(\boldsymbol{q}, \omega) + (s_1 - p) w_{\boldsymbol{n}}(\boldsymbol{q}, \omega). \tag{17}$$

In the K'-system the Maxwell equations have the same form (1) and in the second medium (the half-space $z > d$), the field $\boldsymbol{E}_2'$, $\boldsymbol{B}_2'$ is given by the same formulae (5)–(9) with the x-coordinate changed to x', the index 1 changed to 2, $\cos k_z z$, $\sin k_z z$ replaced by $\cos k_z(z-d)$, $\sin k_z(z-d)$ and a change in the sign of s (the 'reflected' waves now propagate along the positive z-direction). In the space between the media in the K'-system, the field is given by the same formulae, equations (11)–(13), with x changed to x', and $\boldsymbol{v}$, $\boldsymbol{w}$ replaced by

$\boldsymbol{v}'$, $\boldsymbol{w}'$. The relations between the fields in the K- and K'-systems are determined by Lorentz transformation. Neglecting the terms of order $(V/c)^2$, these relations are given by

$$\boldsymbol{v}'(\boldsymbol{q}', \omega') = \boldsymbol{v}(\boldsymbol{q}, \omega) + \frac{V}{\omega}\left[\boldsymbol{e}_x\left[\boldsymbol{k} \times \boldsymbol{v}(\boldsymbol{q}, \omega)\right]\right] \tag{18}$$

$$\boldsymbol{w}'(\boldsymbol{q}', \omega') = \boldsymbol{w}(\boldsymbol{q}, \omega) + \frac{V}{\omega}\left[\boldsymbol{e}_x\left[\widetilde{\boldsymbol{k}} \times \boldsymbol{w}(\boldsymbol{q}, \omega)\right]\right] \tag{19}$$

where $\boldsymbol{k} = (\boldsymbol{q}, p)$, $\widetilde{\boldsymbol{k}} = (\boldsymbol{q}, -p)$, $\omega' = \omega - q_x V$, $\boldsymbol{q}' = \boldsymbol{q} - (V\omega/c^2)\boldsymbol{e}_x$.

In the K'-system the boundary conditions at the surface of body **2** at $z = d$ give the equations

$$\begin{aligned} ps_2^- \int_{-\infty}^{\infty} &(q' g_{2z}(\boldsymbol{q}', k_z, \omega') + s_2^- q_{2\boldsymbol{q}'}(\boldsymbol{q}', k_z, \omega')) \frac{1}{k_z^2 - s_2^{-2}}\, \mathrm{d}k_z \\ &= (p\varepsilon_2^- - s_2^-) v'_{\boldsymbol{q}'}(\boldsymbol{q}', \omega')\mathrm{e}^{\mathrm{i}pd} + (s_2^- + p\varepsilon_2^-) w'_{\boldsymbol{q}'}(\boldsymbol{q}', \omega')\mathrm{e}^{-\mathrm{i}pd} \end{aligned} \tag{20}$$

$$s_2^- \left(\frac{\omega'}{c}\right)^2 \int_{-\infty}^{\infty} \frac{g_{2\boldsymbol{n}'}(\boldsymbol{q}', k_z, \omega')}{k_z^2 - s_2^{-2}}\, \mathrm{d}k_z = (s_2^- - p) v'_{\boldsymbol{n}'}(\boldsymbol{q}', \omega')\mathrm{e}^{\mathrm{i}pd} + (s_2^- + p) w'_{\boldsymbol{n}'}(\boldsymbol{q}', \omega')\mathrm{e}^{-\mathrm{i}pd} \tag{21}$$

where $\varepsilon_2^- = \varepsilon_2(\omega - q_x V)$, and

$$s_2^- = \sqrt{(\omega'/c)^2 \varepsilon_2(\omega') - q'^2} = \sqrt{\frac{(\omega - q_x V)^2}{c^2}(\varepsilon_2(\omega - q_x V) - 1) + p^2}. \tag{22}$$

p is invariant under Lorentz transformation. Now from the equations (18), (19) with accuracy to the terms of the first order in V/c we have

$$v'_{\boldsymbol{q}'}(\boldsymbol{q}', \omega') = (\boldsymbol{v}' \cdot \boldsymbol{e}_{\boldsymbol{q}'}) \approx v_{\boldsymbol{q}}(\boldsymbol{q}, \omega) + \frac{q_y p^2 V}{\omega q^2} v_{\boldsymbol{n}}(\boldsymbol{q}, \omega) \tag{23}$$

$$v'_{\boldsymbol{n}'} = (\boldsymbol{v}' \cdot \boldsymbol{e}_{\boldsymbol{n}'}) \approx \frac{\omega'}{\omega} v_{\boldsymbol{n}} - \frac{\omega q_y V}{c^2 q^2} v_{\boldsymbol{q}}. \tag{24}$$

Similar equations can be written for $w'_{\boldsymbol{q}'}$, $w'_{\boldsymbol{n}'}$. After substituting (23) and (24) into the equations (16), (17) and (20), (21) we get a system of four equations. These equations can be solved considering the second terms in the equations (23), (24) as a small perturbation. In zero order we neglect the second terms. The zero-order solution has the form

$$\begin{aligned} v_{\boldsymbol{q}}^0 = \int_{-\infty}^{\infty} \frac{p}{\Delta} &\left\{ s_1 \mathrm{e}^{-\mathrm{i}pl}(s_2^- + \varepsilon_2^- p) \frac{q g_{1z}(\boldsymbol{q}, k_z, \omega) - s_1 g_{1\boldsymbol{q}}(\boldsymbol{q}, k_z, \omega)}{k_z^2 - s_1^2} \right. \\ &\left. + s_2^-(\varepsilon_1 p - s_1) \frac{q' g_{2z}(\boldsymbol{q}', k_z, \omega') + s_2^- g_{2\boldsymbol{q}'}(\boldsymbol{q}', k_z, \omega')}{k_z^2 - s_2^{-2}} \right\} \mathrm{d}k_z \end{aligned} \tag{25}$$

$$\begin{aligned} w_{\boldsymbol{q}}^0 = \int_{-\infty}^{\infty} \frac{p}{\Delta} &\left\{ -s_1 \mathrm{e}^{\mathrm{i}pd}(\varepsilon_2^- p - s_2^-) \frac{q g_{1z}(\boldsymbol{q}, k_z, \omega) - s_1 g_{1\boldsymbol{q}}(\boldsymbol{q}, k_z, \omega)}{k_z^2 - s_1^2} \right. \\ &\left. - s_2^-(\varepsilon_1 p + s_1) \frac{q' g_{2z}(\boldsymbol{q}', k_z, \omega') + s_2^- g_{2\boldsymbol{q}'}(\boldsymbol{q}', k_z, \omega')}{k_z^2 - s_2^{-2}} \right\} \mathrm{d}k_z \end{aligned} \tag{26}$$

$$v_{\boldsymbol{n}}^0 = \int_{-\infty}^{\infty} \frac{\omega}{c^2 \Delta'} \left\{ -\omega s_1 \mathrm{e}^{-\mathrm{i}pd}(s_2^- + p) \frac{g_{1\boldsymbol{n}}(\boldsymbol{q}, k_z, \omega)}{k_z^2 - s_1^2} + \omega' s_2^-(s_1 - p) \frac{g_{2\boldsymbol{n}'}(\boldsymbol{q}', k_z, \omega')}{k_z^2 - s_2^{-2}} \right\} \mathrm{d}k_z \tag{27}$$

$$w_{\boldsymbol{n}}^0 = \int_{-\infty}^{\infty} \frac{\omega}{c^2\Delta'} \left\{ \omega s_1 \mathrm{e}^{\mathrm{i}pd}(s_2^- - p)\frac{g_{1n}(\boldsymbol{q}, k_z, \omega)}{k_z^2 - s_1^2} - \omega' s_2^-(s_1 + p)\frac{g_{2n'}(\boldsymbol{q}', k_z, \omega')}{k_z^2 - s_2^{-2}} \right\} \, \mathrm{d}k_z \tag{28}$$

$$v_z = -\frac{q v_{\boldsymbol{q}}}{p} \qquad w_z = \frac{q w_{\boldsymbol{q}}}{p} \tag{29}$$

where we have introduced the notation

$$\Delta = \mathrm{e}^{\mathrm{i}pd}(\varepsilon_1 p - s_1)(\varepsilon_2^- p - s_2^-) - \mathrm{e}^{-\mathrm{i}pd}(\varepsilon_1 p + s_1)(\varepsilon_2^- p + s_2^-)$$
$$\Delta' = \mathrm{e}^{\mathrm{i}pd}(s_1 - p)(s_2^- - p) - \mathrm{e}^{-\mathrm{i}pd}(p + s_1)(p + s_2^-).$$

The first-order solution has the form

$$v_{\boldsymbol{q}}^1 = \frac{(p\varepsilon_1 - s_1)\Lambda}{\Delta} \qquad w_{\boldsymbol{q}}^1 = -\frac{(p\varepsilon_1 + s_1)\Lambda}{\Delta} \tag{30}$$

$$v_{\boldsymbol{n}}^1 = \frac{(s_1 - p)\Lambda'}{\Delta'} \qquad w_{\boldsymbol{n}}^1 = -\frac{(p + s_1)\Lambda'}{\Delta'} \tag{31}$$

where

$$\Lambda = -\frac{q_y p^2 V}{\omega q^2}\left[(p\varepsilon_2^- - s_2^-)v_{\boldsymbol{n}}^0 \mathrm{e}^{\mathrm{i}pd} + (p\varepsilon_2^- + s_2^-)w_{\boldsymbol{n}}^0 \mathrm{e}^{-\mathrm{i}pd}\right]$$
$$\Lambda' = \frac{\omega' q_y V}{c^2 q}\left[(s_2^- - p)v_{\boldsymbol{q}}^0 \mathrm{e}^{\mathrm{i}pd} + (s_2^- + p)w_{\boldsymbol{q}}^0 \mathrm{e}^{-\mathrm{i}pd}\right].$$

3. Calculation of the force of friction

The frictional stresses σ and $-\sigma$ which act on the surfaces of the two bodies can be obtained from the xz-component of the Maxwell stress tensor σ_{ij}, evaluated at $z = 0$:

$$\sigma = \frac{1}{8\pi}\int_{-\infty}^{+\infty} \mathrm{d}\omega \, [\langle E_{3z}E_{3x}^*\rangle + \langle E_{3z}^* E_{3x}\rangle + \langle B_{3z}B_{3x}^*\rangle + \langle B_{3z}^* B_{3x}\rangle]_{z=0}. \tag{32}$$

Here the $\langle\cdots\rangle$ denote statistical averaging over the random field. The averaging is carried out with the aid of equation (4). Note that the components of the random field $\boldsymbol{g}_1$ and $\boldsymbol{g}_2$ referring to different media are statistically independent, so the average of their product is zero. Writing the squares of the integrals (11), (12) in the usual way as double integrals, and carrying out one integration over the δ-function, we obtain

$$\sigma = \frac{1}{8\pi}\int \mathrm{d}\omega \, \mathrm{d}^2 q \, [\langle E_{3z}(\boldsymbol{q}, \omega)E_{3x}^*(\boldsymbol{q}, \omega)\rangle + \langle E_{3z}^*\boldsymbol{q}, \omega)E_{3x}(\boldsymbol{q}, \omega)\rangle$$
$$+ \langle B_{3z}(\boldsymbol{q}, \omega)B_{3x}^*(\boldsymbol{q}, \omega) + \langle B_{3z}^*(\boldsymbol{q}, \omega)B_{3x}(\boldsymbol{q}, \omega)\rangle]_{z=0} \tag{33}$$

where one must substitute, in place of $\boldsymbol{E}_3$ and $\boldsymbol{B}_3$, the expressions in the integrands (11), (12) determined by the formulae (25)–(31), and the average product $\langle g_i(\boldsymbol{k}, \omega)g_k^*(\boldsymbol{k}, \omega)\rangle$ is to be taken as $(1/2 + n(\omega))\varepsilon''(\omega)\delta_{ik}/\pi^3$. For a given value of $\boldsymbol{q}$ it is convenient to express the components E_x and B_x in terms of the components along the two vectors $\boldsymbol{e_q}$ and $\boldsymbol{e_n}$:

$$E_x = (q_x/q)E_{\boldsymbol{q}} - (q_y/q)E_{\boldsymbol{n}}$$
$$B_x = (q_x/q)B_{\boldsymbol{q}} - (q_y/q)B_{\boldsymbol{n}}.$$

Thus we can write

$$\sigma = \frac{1}{8\pi}\int \mathrm{d}\omega\, \mathrm{d}^2q \left\{ \frac{q_x}{q}[\langle E_z(\boldsymbol{q},\omega)E^*_{\boldsymbol{q}}(\boldsymbol{q},\omega)\rangle + \langle E^*_z q,\omega)E_{\boldsymbol{q}}(\boldsymbol{q},\omega)\rangle \right.$$
$$+ \langle B_z(\boldsymbol{q},\omega)B^*_{\boldsymbol{q}}(\boldsymbol{q},\omega)\rangle + \langle B^*_z(\boldsymbol{q},\omega)B_{\boldsymbol{q}}(\boldsymbol{q},\omega)\rangle]_{z=0}$$
$$- \frac{q_y}{q}[\langle E_z(\boldsymbol{q},\omega)E^*_{\boldsymbol{n}}(\boldsymbol{q},\omega)\rangle + \langle E^*_z q,\omega)E_{\boldsymbol{n}}(\boldsymbol{q},\omega)\rangle$$
$$\left. + \langle B_z(\boldsymbol{q},\omega)B^*_{\boldsymbol{n}}(\boldsymbol{q},\omega)\rangle + \langle B^*_z(\boldsymbol{q},\omega)B_{\boldsymbol{n}}(\boldsymbol{q},\omega)\rangle]_{z=0}\right\}. \tag{34}$$

According to equations (11), (12)

$$\begin{aligned} E_z &= (v_z + w_z) = (q/p)(w_{\boldsymbol{q}} - v_{\boldsymbol{q}}) = (qp^*/|p|^2)(w_{\boldsymbol{q}} - v_{\boldsymbol{q}}) \\ E_{\boldsymbol{q}} &= v_{\boldsymbol{q}} + w_{\boldsymbol{q}} \\ E_{\boldsymbol{n}} &= v_{\boldsymbol{n}} + w_{\boldsymbol{n}} \\ B_z &= (cq/\omega)(v_{\boldsymbol{n}} + w_{\boldsymbol{n}}) \\ B_{\boldsymbol{q}} &= (cp/\omega)(w_{\boldsymbol{n}} - v_{\boldsymbol{n}}) \\ B_{\boldsymbol{n}} &= (\omega p^*/c|p|^2)(v_{\boldsymbol{q}} - w_{\boldsymbol{q}}). \end{aligned} \tag{35}$$

After substituting these expressions into formula (34) one can see that the second term is identically equal to zero. From equations (25)–(31) it follows that the zero- and first-order solutions are statistically independent; then, neglecting the terms of order $(v/c)^2$, from (34), (35) we obtain

$$\sigma = \frac{1}{4\pi}\int_0^{+\infty} \mathrm{d}\omega \int \mathrm{d}^2q\; q_x \left(\left[\frac{1}{|p|^2}(p+p^*)(\langle|w^0_{\boldsymbol{q}}|^2\rangle - \langle|v^0_{\boldsymbol{q}}|^2\rangle) \right.\right.$$
$$\left. + (p-p^*)\langle(v^0_{\boldsymbol{q}}w^{0*}_{\boldsymbol{q}} - v^{0*}_{\boldsymbol{q}}w^0_{\boldsymbol{q}})\rangle \right]$$
$$\left. + \left(\frac{c}{\omega}\right)^2 \left[(p+p^*)(\langle|w^0_{\boldsymbol{n}}|^2\rangle - \langle|v^0_{\boldsymbol{n}}|^2\rangle) - (p-p^*)\langle(v^0_{\boldsymbol{n}}w^{0*}_{\boldsymbol{n}} - v^{0*}_{\boldsymbol{n}}w^0_{\boldsymbol{n}})\rangle \right] \right) \tag{36}$$

where we change the integration over ω between the limits $-\infty$ and $+\infty$ to integration only over positive values of ω, which gives the extra factor two.

Taking into account that $p = p^*$ for $q < \omega/c$ and $p = -p^*$ for $q > \omega/c$, and carrying out the integration over $\mathrm{d}k_z$ with the help of the formula

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}k_z}{|k_z^2 - s^2|^2} = \frac{\mathrm{i}\pi}{|s|^2\,(s - s^*)}$$

we obtain, after substituting (25)–(31) into (36),

$$\sigma = \frac{\hbar}{8\pi^3}\int_0^{\infty} \mathrm{d}\omega \int_{q<\omega/c} \mathrm{d}^2q\; q_x$$
$$\times \left\{ \frac{(1-|R_{1p}|^2)(1-|R^-_{2p}|^2)}{|1-\mathrm{e}^{2\mathrm{i}pd}R_{1p}R^-_{2p}|^2}(n(\omega - q_xV) - n(\omega)) + [R_p \to R_s] \right\}$$
$$+ \frac{\hbar}{2\pi^3}\int_0^{\infty} \mathrm{d}\omega \int_{q>\omega/c} \mathrm{d}^2q\; q_x \mathrm{e}^{-2|p|d}$$
$$\times \left\{ \frac{\mathrm{Im}\, R_{1p}\, \mathrm{Im}\, R^-_{2p}}{|1-\mathrm{e}^{-2|p|d}R_{1p}R^-_{2p}|^2}(n(\omega - q_xV) - n(\omega)) + [R_p \to R_s] \right\} \tag{37}$$

where

$$R_{ip} = \frac{\varepsilon_i p - s_i}{\varepsilon_i p + s_i} \qquad R_{is} = \frac{\varepsilon_i - s_i}{\varepsilon_i + s_i}$$

$$R_{ip}^{\pm} = \frac{\varepsilon_i^{\pm} p - s_i^{\pm}}{\varepsilon_i^{\pm} p + s_i^{\pm}} \qquad R_{is}^{\pm} = \frac{\varepsilon_i^{\pm} - s_i}{\varepsilon_i^{\pm} + s_i}.$$

$\varepsilon_i^{\pm}(\omega) = \varepsilon_i(\omega \pm q_x V)$ and $s_i^{\pm}(\omega) = s_i(\omega \pm q_x V)$, $i = 1, 2$. Note that R_{ip} and R_{is} are the electromagnetic reflection factors for p-polarized and s-polarized light, respectively. (p-polarized light has the electric field vector in the plane of incidence while the electric field vector is perpendicular to this plane for s-polarized light.) The first term in (37) is the contribution to the friction force from the propagating (radiating) electromagnetic field, i.e., the black-body radiation. This term includes only the thermal radiation and is equal to zero at $T = 0$. The second term is derived from the evanescent field, i.e., from the component of the electromagnetic field which decays exponentially with the distance away from the surfaces of the bodies. This term does not vanish even at $T = 0$ K because of quantum fluctuations in the charge density in the solids.

4. Some limiting cases

Let us first consider distances $d \ll c/\omega_p$, where ω_p is the plasma frequency of the metals. For typical metals, $c/\omega_p \approx 200$ Å. In this case the main contribution comes from $q \gg \omega_p/c$, and we have $s_1 \approx s_2 \approx p \approx \mathrm{i}q$, $R_s \approx 0$ and

$$R_p \approx \frac{\varepsilon - 1}{\varepsilon + 1}.$$

In this approximation the integration over q can be extended to the whole q-plane. Using these approximations, the second term in (11) can be written as

$$\begin{aligned}
\sigma = \frac{\hbar}{4\pi^3}\int_0^\infty \mathrm{d}\omega \int \mathrm{d}^2q\; q_x \mathrm{e}^{-2|p|d}\Bigg\{\Bigg(&\frac{\operatorname{Im} R_{1p}\operatorname{Im} R_{2p}^-}{|1-\mathrm{e}^{-2|p|d}R_{1p}R_{2p}^-|^2} + (1 \leftrightarrow 2)\Bigg) \\
&\times (n(\omega - q_x V) - n(\omega)) + [R_p \to R_s]\Bigg\} \\
= \frac{\hbar}{2\pi^3}\int_{-\infty}^{\infty}\mathrm{d}q_y \int_0^\infty \mathrm{d}q_x\; q_x \mathrm{e}^{-2qd}&\left\{\int_0^\infty \mathrm{d}\omega\,[n(\omega) - n(\omega + q_x v)]\right. \\
&\times \Bigg(\frac{\operatorname{Im} R_{1p}^{+}\operatorname{Im} R_{2p}}{|1-\mathrm{e}^{-2|p|d}R_{1p}^{+}R_{2p}|^2} + (1 \leftrightarrow 2)\Bigg) \\
&\left. - \int_0^{q_x v}\mathrm{d}\omega\,[n(\omega) + 1/2]\Bigg(\frac{\operatorname{Im} R_{1p}^{-}\operatorname{Im} R_{2p}}{|1-\mathrm{e}^{-2qd}R_{1p}^{-}R_{2p}|^2} + (1 \leftrightarrow 2)\Bigg)\right\}
\end{aligned} \tag{38}$$

where we have used the relation $n(-\omega) = -n(\omega) - 1$. At zero temperature the Bose–Einstein factor $n(\omega) = 0$, and only the second term in (38) will contribute to the sliding friction; in this limit our expression for the friction force for two identical solids is in agreement with that of Pendry. In appendix A we show that the zero-temperature result can be generalized to include nonlocal optics effects, by replacing the reflection factor $R_p(\omega)$ in (38) by the surface response function $g(q, \omega)$. Next, let us consider the limiting case of low sliding velocity or high temperature, namely, $V \ll cd/d_W$, where $d_W = c\hbar/k_B T$ is the Wien length (typically $d_W \approx 10^5$ Å). In this case

$$n(\omega) - n(\omega + q_x V) \approx -q_x V\frac{\mathrm{d}n}{\mathrm{d}\omega} = \frac{\mathrm{e}^{\hbar\omega/k_B T}}{(\mathrm{e}^{\hbar\omega/k_B T} - 1)^2}\frac{\hbar q_x V}{k_B T}$$

and in the second term in (38) we can put

$$n(\omega) \approx k_B T/\hbar\omega.$$

Substituting these results in (38) gives

$$\sigma = \frac{\hbar V}{2\pi^2}\int_0^\infty \mathrm{d}q\, q^3 \mathrm{e}^{-2qd}\int_0^\infty \mathrm{d}\omega\left(-\frac{\mathrm{d}n}{\mathrm{d}\omega}\right)\frac{\operatorname{Im}R_{1p}\operatorname{Im}R_{2p}}{|1-\mathrm{e}^{-2qd}R_{1p}R_{2p}|^2}$$
$$+\frac{2}{\pi^3}k_B T\int_{-\infty}^{\infty}\mathrm{d}q_y\int_0^\infty \mathrm{d}q_x\, q_x \mathrm{e}^{-2qd}\int_0^{q_x V}\frac{\mathrm{d}\omega}{\omega}\,\frac{\operatorname{Im}R_{1p}\operatorname{Im}R_{2p}}{|1-\mathrm{e}^{-2qd}R_{1p}R_{2p}|^2}. \tag{39}$$

The second term in this expression is proportional to V^2 as $V \to 0$ (see below) and can be neglected in the limit of small V. The first term is $\sim V$ and is in agreement with the result obtained by Persson and Zhang if one assumes local optics (which implies making the replacement $g(q,\omega) \to R_p(\omega)$ in [2]). For free-electron-like metals the local optics is accurate if $d \gg l$, where l is the electron mean free path in the metal. If this condition is not satisfied, the general formula of Persson and Zhang must be used.

Let us consider two identical metals described by the dielectric function

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + \mathrm{i}/\tau)} \tag{40}$$

where τ is the relaxation time and ω_p the plasma frequency. Thus, for small frequencies

$$\operatorname{Im}R_p \approx \frac{2\omega}{\omega_p^2\tau} \qquad \operatorname{Re}R_p \approx 1$$

and if we neglect the imaginary part of R_p in the denominator of the integrand in (13) we obtain

$$\sigma = \xi\left(\frac{k_B T}{\hbar\omega_p}\right)^2\frac{1}{(\omega_p\tau)^2}\frac{\hbar V}{d^4} + \frac{2\pi}{45}\frac{k_B T}{\hbar\omega_p}\frac{1}{(\omega_p\tau)^2}\frac{\hbar V^2}{\omega_p d^5} \tag{41}$$

where

$$\xi = \frac{1}{8}\int_0^\infty \frac{\mathrm{d}x\, x^2}{\mathrm{e}^x - 1} \approx 0.5986.$$

In deriving (41) we have used the following standard integrals:

$$\int_0^\infty \frac{\mathrm{d}x\, x}{\mathrm{e}^x - 1} = \frac{\pi^2}{6} \qquad \int_0^\infty \frac{\mathrm{d}x\, x^3}{\mathrm{e}^x - 1} = \frac{\pi^4}{15}.$$

The ratio between the second and first term in (41) equals $\approx (V/c)(d_W/d)$, and in deriving (41) we have assumed that this quantity is much smaller than unity. As an example, if $d = 10$ Å and $V = 1$ m s^{-1} then for typical metals at room temperature ($k_B T \approx 0.025$ eV, $\omega_p\tau \approx 100$, $\hbar\omega_p \approx 10$ eV) the first and the second terms in (41) give $\sigma \approx 10^{-8}$ and $\approx 10^{-13}$ N m^{-2}, respectively.

On the other hand, if $(V/c)(d_W/d) \gg 1$ we get

$$\sigma = \frac{\xi}{2}\left(\frac{k_B T}{\hbar\omega_p}\right)^2\frac{1}{(\omega_p\tau)^2}\frac{\hbar V}{d^4} + \zeta\frac{1}{(\omega_p\tau)^2}\frac{\hbar V}{d^4}\left(\frac{V}{d\omega_p}\right)^2 \tag{42}$$

where

$$\zeta = \frac{5}{2^9\pi^2}\int_0^\infty \frac{\mathrm{d}x\, x^4}{\mathrm{e}^x - 1} = 0.024\,610.$$

The ratio of the second and first terms in (42) equals $\sim 0.1(V/c)^2(d_W/d)^2$. It is clear that at low temperature or high velocities, the second term in (42) will dominate.

Next, let us consider the sliding friction to linear order in the sliding velocity when $c/\omega_p \ll d \ll d_W$. There will be two contributions associated with R_p and R_s. As shown in appendix B, the contribution from R_p is

$$\sigma_p \approx \frac{3\xi}{\pi^2}\frac{\hbar V}{d^4}\left(\frac{d}{d_W}\right)^2 \frac{k_B T}{\hbar\omega_p}\frac{1}{\omega_p \tau}\left(1+\frac{1}{\mathrm{e}}+\ln\frac{d_W}{d}\right). \tag{43}$$

The contribution σ_s from the term involving R_s is given by (see appendix B)

$$\sigma_s \approx C\omega_p \tau \frac{V}{c}\frac{\hbar\omega_p}{d^2 d_W} \tag{44}$$

where $C \approx 0.394$. Comparing (43) with (44) we obtain

$$\sigma_s/\sigma_p \approx (\omega_p\tau)^2(\hbar\omega_p/k_B T)^2.$$

For typical metals at room temperature, $\hbar\omega_p/k_B T \sim 10^3$ and $\omega_p\tau \sim 100$, so $\sigma_s/\sigma_p \sim 10^{10}$, i.e., the main contribution comes from the term involving R_s. As an illustration, if $d = 10^4$ Å and $V = 1$ m s^{-1} then for metals at room temperature, characterized by the same parameter values as were used above, one get $\sigma_s \approx 10^{-8}$ N m^{-2}.

Now, let us consider the radiative contribution to the friction force, which is given by the first term in (38). In linear order in the sliding velocity we get

$$\sigma_{rad} = \frac{\hbar V}{8\pi^3}\int_0^\infty \mathrm{d}\omega \int_{q<\omega/c} \mathrm{d}^2q\; q_x^2 \left\{\frac{(1-|R_{1p}|^2)(1-|R_{2p}|^2)}{|1-\mathrm{e}^{2\mathrm{i}pd}R_{1p}R_{2p}|^2}\left(-\frac{\partial n(\omega)}{\partial\omega}\right)+[R_p \to R_s]\right\}. \tag{45}$$

For separations d much smaller than the Wien wavelength $d_W = c\hbar/k_B T$ we can put $\exp(\mathrm{i}pd) \approx 1$. In this case and for the small frequencies, when $\omega \leqslant k_B T/\hbar \ll 1/\tau$, we get for identical metals described by the dielectric function (40)

$$\frac{(1-|R_p|^2)^2}{|1-R_p^2|^2} = \frac{1}{2} + \frac{(\varepsilon^* s)^2 + (\varepsilon s^*)^2}{4|\epsilon s|^2} \approx \frac{1}{2} + \frac{\varepsilon^* + \varepsilon}{4|\varepsilon|} \approx \frac{1}{2}\left(1+\frac{\omega\tau}{2}\right) \approx \frac{1}{2} \tag{46}$$

and the same result is obtained when R_p in (46) is replaced by R_s. The final result for the radiative friction force has the form

$$\sigma_{rad} = \frac{\hbar V}{8\pi^2 c^4}\int_0^\infty \mathrm{d}\omega\, \omega^3 n(\omega) = \frac{\pi^2}{120}\hbar V\left(\frac{k_B T}{\hbar c}\right)^4. \tag{47}$$

Note that the radiative stress does not depend on the separation and is proportional to T^4. The latter result is, of course, only valid as long as d is small compared with the lateral extent (or linear size) L of the bodies. When d becomes comparable with or larger than L, the friction force between the two bodies will decrease monotonically with increasing d. At room temperature and at the sliding velocity $V = 1$ m s^{-1} one gets $\sigma_{rad} \approx 10^{-15}$ N m^{-2}. The ratio of this contribution to σ_s from (44) is

$$\frac{\sigma_{rad}}{\sigma_s} = 0.1\left(\frac{d}{d_W}\right)^2 \frac{k_B T}{\hbar\omega_p}\frac{1}{\omega_p\tau} \sim 10^{-6}\left(\frac{d}{d_W}\right)^2.$$

Thus for $d \sim d_W$ (which is of order $\sim 10^5$ for typical metals at room temperature) the nonradiative part dominates over the radiative contribution by a factor $\sim 10^6$. However, for large enough distances the radiative part dominates as this contribution is finite for arbitrary separations d.

5. Summary and conclusions

We have calculated the friction force between two arbitrary bodies with flat surfaces separated by a vacuum slab of thickness d, and moving with a relative velocity V. The separation d is assumed to be so large that the only interaction between the bodies is via the electromagnetic field associated with *thermal* or *quantum* fluctuations in the solids. A general formula for the friction force has been obtained, which is valid for arbitrary velocity V, separation d and temperature T, and applicable to any bodies. At low sliding velocity only thermal fluctuations give a contribution to the friction force, linearly proportional to the velocity V. Quantum fluctuations give a nonlinear (in V) contribution to the friction force which is usually negligible compared with the thermal contribution. (There is also a contribution from quantum fluctuations which is proportional to V, resulting from electron–photon processes of higher order than those considered in our work. However, this contribution decays as $\exp(-2Gd)$ (where $G = 2\pi/a$ is the smallest reciprocal-lattice vector) and is negligibly small already for $d = 10$ Å; see [2, 11].) We have studied the detailed distance dependence of the friction force from short distances, where retardation effects can be neglected, to large distances where retardation effects and black-body radiation are important. In most practical cases, involving sliding of a block on a substrate, the van der Waals friction makes a negligible contribution to the friction force (the main part of the friction arises from the regions of real contact between the solids). However, in some special cases the van der Waals friction is very important [2]. For example, quantum fluctuations contribute in an important manner to the friction force acting on thin physisorbed layers of atoms sliding on metallic surfaces [12]. (In this case there is an overlap of the wavefunctions of the sliding layer and those of the metal, which results in a second contribution to the friction force, derived from the repulsive 'contact interaction' (Pauli repulsion) between the sliding layer and the substrate.) In addition, the contribution from thermal fluctuations gives the dominating drag force in some experiments involving parallel 2D electron systems [13].

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Appendix A

In this appendix we derive an expression for the (nonlinear) sliding friction using a nonlocal optics description of the metals. For simplicity, we focus on zero temperature, $T = 0$ K, and assume that d is so short that retardation effects can be neglected (only in this 'short-distance' regime will nonlocal effects be important). The calculation is based on the formalism developed in [2, 14]. Let us first define the linear response function $g(q, \omega)$ which is needed below. Assume that a semi-infinite metal occupies the half-space $z \leqslant 0$. A charge distribution in the half-space $z > d$ gives rise to an (external) potential which must satisfy the Laplace equation for $z < d$ and which therefore can be written as a sum of evanescent plane waves of the form

$$\phi_{ext} = \phi_0 e^{qz} e^{i\boldsymbol{q}\cdot\boldsymbol{x} - i\omega t}$$

where $\boldsymbol{q} = (q_x, q_y)$ is a 2D wavevector. This potential will induce a charge distribution in the solid (occupying $z < 0$) which in turn gives rise to an electric potential which must satisfy the Laplace equation for $z > 0$, and which therefore can be expanded into evanescent plane waves which decay with increasing $z > 0$. Thus the total potential for $0 < z < d$ can be expanded in functions of the form

$$\phi_{ext} = \phi_0(\mathrm{e}^{qz} - g\mathrm{e}^{-qz})\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x}-\mathrm{i}\omega t}$$

where the reflection factor $g = g(q, \omega)$. For the present purposes, we can treat the low-energy electron–hole pair excitations in the metals as bosons. As shown in reference [14], the Hamiltonian for the total system can be written as

$$H = \sum_{\boldsymbol{q}\alpha_1} \hbar\omega_{\boldsymbol{q}\alpha_1} b^+_{\boldsymbol{q}\alpha_1} b_{\boldsymbol{q}\alpha_1} + \sum_{\boldsymbol{q}\alpha_2} \hbar\omega_{\boldsymbol{q}\alpha_2} b^+_{\boldsymbol{q}\alpha_2} b_{\boldsymbol{q}\alpha_2} + \hbar\omega b^+ b$$
$$+ \sum_{\boldsymbol{q}\alpha_1 n} C_{q\alpha_1} \mathrm{e}^{-qz_n}(b_{\boldsymbol{q}\alpha_1}\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot(\boldsymbol{x}_n+\boldsymbol{V}t)} + \mathrm{h.c.}). \quad \text{(A1)}$$

Here $\omega_{\boldsymbol{q}\alpha_1}$, $b^+_{\boldsymbol{q}\alpha_1}$ and $b_{\boldsymbol{q}\alpha_1}$ are the angular frequency and creation and annihilation operators for the bosons (of solid **1**) with the quantum numbers $(\boldsymbol{q}, \alpha_1)$, and $C_{\boldsymbol{q}\alpha_1}$ are parameters determining the coupling between the boson excitations in solid **1** with the electrons in solid **2**. Similarly, $b^+_{\boldsymbol{q}\alpha_2}$ and $b_{\boldsymbol{q}\alpha_2}$ are creation and annihilation operators for bosons in solid **2**, and $(\boldsymbol{x}_n, \boldsymbol{z}_n)$ is the position operator of electron n in solid **2**, which in principle could be expressed in terms of the operators $b^+_{\boldsymbol{q}\alpha_2}$ and $b_{\boldsymbol{q}\alpha_2}$, but for the present purpose this is not necessary. As shown in [14], $C_{\boldsymbol{q}\alpha_1}$ can be related to Im $g_1(q, \omega)$ via

$$\sum_{\alpha_1} |C_{\boldsymbol{q}\alpha_1}|^2 \delta(\omega - \omega_{q\alpha_1}) = \frac{2e^2\hbar}{Aq} \operatorname{Im} g_1(q, \omega). \quad \text{(A2)}$$

We can write the Hamiltonian of the interaction between solids **1** and **2** as

$$H' = \sum_{\boldsymbol{q}} (\hat{V}_{\boldsymbol{q}} \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{V}t} + \mathrm{h.c.}).$$

Using time-dependent perturbation theory (with H' as the perturbation) we can calculate the energy transfer from the translational motion (kinetic energy) to internal excitations in the solids (boson excitations $\omega_{\boldsymbol{q}\alpha_1}$ and $\omega_{\boldsymbol{q}\alpha_2}$ in solids **1** and **2**, respectively):

$$P = \frac{2\pi}{\hbar^2} \sum_{\boldsymbol{q}\alpha_1\alpha_2} \hbar\omega_{\boldsymbol{q}} \delta(\omega_{\boldsymbol{q}} - \omega_{\boldsymbol{q}\alpha_2} - \omega_{\boldsymbol{q}\alpha_1}) |C_{q\alpha_1}|^2$$
$$\times\, \mathrm{e}^{-2qd} \left| \langle n_{\boldsymbol{q}\alpha_1} = 1, n_{\boldsymbol{q}\alpha_2} = 1 | \sum_n \mathrm{e}^{-q(z_n-d)} \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x}_n} b^+_{\boldsymbol{q}\alpha_1} |0, 0\rangle \right|^2 \quad \text{(A3)}$$

where $\omega_{\boldsymbol{q}} = |\boldsymbol{q} \cdot \boldsymbol{V}|$. To simplify (A3), let us write

$$\delta(\omega_{\boldsymbol{q}} - \omega_{\boldsymbol{q}\alpha_1} - \omega_{\boldsymbol{q}\alpha_2}) = \int \mathrm{d}\omega'\, \delta(\omega' - \omega_{\boldsymbol{q}\alpha_1}) \delta(\omega_{\boldsymbol{q}} - \omega' - \omega_{\boldsymbol{q}\alpha_2}). \quad \text{(A4)}$$

Substituting (A4) in (A3) and using (A2) gives

$$P = \frac{4\pi e^2}{A} \sum_{\boldsymbol{q}} \frac{\omega_{\boldsymbol{q}}}{q} \mathrm{e}^{-2qd} \int \mathrm{d}\omega' \operatorname{Im} g_1(q, \omega') M_q(\omega_{\boldsymbol{q}} - \omega') \quad \text{(A5)}$$

where

$$M_q(\omega) = \sum_{\alpha_2} \delta(\omega - \omega_{\boldsymbol{q}\alpha_2}) \left| \langle n_{\boldsymbol{q}\alpha_2} = 1 | \sum_n \mathrm{e}^{-q(z_n-d)} \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x}_n} |0\rangle \right|^2.$$

But it has been shown elsewhere that [15]

$$\frac{A\hbar q}{2\pi^2 e^2}\,\mathrm{Im}\,g_2(q,\omega) = \sum_{\alpha_2} \delta(\omega - \omega_{q\alpha_2}) \left| \langle n_{q\alpha_2} = 1 | \sum_n \mathrm{e}^{-q(z_n - d)} \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x}_n} |0\rangle \right|^2$$

so we have that

$$M_q(\omega) = \frac{A\hbar q}{2\pi^2 e^2}\,\mathrm{Im}\,g_2(q,\omega). \tag{A6}$$

Substituting this result in (A5) gives

$$P = \frac{2\hbar}{\pi} \sum_{\boldsymbol{q}} \omega_{\boldsymbol{q}} \mathrm{e}^{-2qd} \int \mathrm{d}\omega'\,\mathrm{Im}\,g_1(q,\omega')\,\mathrm{Im}\,g_2(q,\omega_{\boldsymbol{q}} - \omega'). \tag{A7}$$

Finally, making the replacement

$$\sum_{\boldsymbol{q}} \to \frac{A}{4\pi^2} \int \mathrm{d}^2 q$$

and using the relation $P = \sigma A V$ between the power P and the shear stress σ gives

$$\sigma = \frac{\hbar}{2\pi^3} \int \mathrm{d}^2 q\; |q_x| \mathrm{e}^{-2qd} \int_0^{|q_x|V} \mathrm{d}\omega'\,\mathrm{Im}\,g_1(q,\omega')'\,\mathrm{Im}\,g_2(q,|q_x|V - \omega') \tag{A8}$$

where we have used that $\mathrm{Im}\,g(q,\omega) = 0$ for $\omega < 0$. The coupling H' gives rise not only to real excitations but also to screening (image charge effects). To take these into account one must go to higher order in perturbation theory. Following reference [2] this gives a modification of (A8):

$$\sigma = \frac{\hbar}{2\pi^3} \int \mathrm{d}^2 q\; \frac{|q_x| \mathrm{e}^{-2qd}}{|1 - g_1(q,0) g_2(q,0) \mathrm{e}^{-2qd}|^2} \int_0^{|q_x|V} \mathrm{d}\omega'\,\mathrm{Im}\,g_1(q,\omega')\,\mathrm{Im}\,g_2(q,|q_x|V - \omega') \tag{A9}$$

where we have assumed that the small frequencies involved in the real excitations are screened in an adiabatic manner so that g_1 and g_2 can be evaluated at zero frequency in the screening factor. In the case of local optics, this expression for σ agrees with the last term in (38) evaluated at zero temperature. At finite temperature ($T > 0$ K) an extra factor of $[1 + 2n(\omega')]$ must be inserted in the frequency integral in (A9) to take into account the enhanced probability for excitation of bosons at finite temperature. For $T > 0$ K one must, in addition to the process considered above, also include scattering processes where a thermally excited boson is annihilated either in solid **1** or in solid **2**, namely $(n_{q\alpha_1} = 0, n_{q\alpha_2} = 1) \to (1,0)$ and $(1,0) \to (0,1)$. These processes were considered in reference [2] and give, in the local optics case, the frictional stress corresponding to the first term in (38).

Appendix B

In this appendix we calculate the sliding friction to linear order in the sliding velocity when $c/\omega_p \ll d \ll d_W$ (where $d_W = c\hbar/k_B T$). In this case the main contribution σ comes from the first term in (39) to which we must add a similar term involving R_s. In this integral we replace the integration variable q with $\bar{p} = 2dq$. The integral over $\bar{p}$ is divided into two parts: the integral over $(0, \bar{p}_0)$ and that over $(\bar{p}_0, \infty)$, where $p_0 \sim dk_B T/c\hbar \ll 1$. In the integral over $(\bar{p}_0, \infty)$ and for $\omega > \omega_0$, where

$$\omega_0 \sim \left(\frac{c}{\omega_p d}\right)^2 \frac{1}{\tau}$$

we can put

$$\mathrm{Im}\, R_p \approx \frac{2}{\bar{p}}\left(\frac{2\omega d}{c}\right)\left(\frac{\omega}{2\omega_p^2\tau}\right)^{1/2} \tag{B1}$$

$$\mathrm{Im}\, R_s \approx -\frac{2\bar{p}c}{\omega_p d}\left(\frac{1}{\omega\tau}\right)^{1/2}. \tag{B2}$$

Let us first consider the contribution σ_p to (39) from terms involving R_p. We then obtain

$$\begin{aligned}\sigma_p &= \frac{\hbar v}{\pi^2}\left(\frac{1}{2d}\right)^4 \int_{\bar{p}_0}^{\infty} \mathrm{d}\bar{p}\; \bar{p}\, \frac{\mathrm{d}}{\mathrm{d}\bar{p}}\left(-\frac{1}{\mathrm{e}^{\bar{p}}-1}\right) \int_0^{\infty} \mathrm{d}\omega\; \frac{\omega}{2\omega_p^2\tau}\left(\frac{2\omega d}{c}\right)^2\left(-\frac{\mathrm{d}n}{\mathrm{d}\omega}\right)\\
&\approx \frac{3}{8\pi^2}\frac{\hbar v}{d^4}\left(1+\int_{\bar{p}_0}^{1}\mathrm{d}\bar{p}\;\frac{1}{\mathrm{e}^{\bar{p}}-1}+\int_1^{\infty}\mathrm{d}\bar{p}\;\mathrm{e}^{-\bar{p}}\right)\\
&\quad\times\left(\frac{k_B T d}{\hbar c}\right)^2\frac{k_B T}{\hbar\omega_p}\frac{1}{\omega_p\tau}\int_0^{\infty}\frac{\mathrm{d}x\; x^2}{\mathrm{e}^x-1}\\
&\approx \frac{3\xi}{\pi^2}\frac{\hbar v}{d^4}\left(\frac{k_B T d}{\hbar c}\right)^2\frac{k_B T}{\hbar\omega_p}\frac{1}{\omega_p\tau}\left(1+\frac{1}{\mathrm{e}}+\ln\frac{d_w}{d}\right)\end{aligned} \tag{B3}$$

where $\xi = 0.5986$ (see section 3). The integral over $0 < \bar{p} < \bar{p}_0$ can be shown to give a negligible contribution to the linear (in the sliding velocity) friction force. This follows from the following equation:

$$\frac{\mathrm{d}}{\mathrm{d}\omega}\frac{(\mathrm{Im}\, R_p)^2}{|1-\mathrm{e}^{-\bar{p}}R_pR_p|^2} \approx \frac{\mathrm{d}}{\mathrm{d}\omega}\frac{(\mathrm{Im}\, R_p)^2}{|1-R_pR_p|^2} = \frac{\mathrm{d}}{\mathrm{d}\omega}\left(\frac{(\mathrm{Im}(s/\varepsilon))^2}{|s/\varepsilon|^2}\right) \approx 0$$

where we have used that $\mathrm{Im}(s/\epsilon)/|s/\epsilon|$ is approximately independent of frequency. Next, let us consider the contribution σ_s to (39) from the term involving R_s. We get

$$\begin{aligned}\sigma_s &\approx \frac{5}{16\pi^2}\frac{\hbar v}{d^4}\frac{\hbar}{k_B T\tau}\left(\frac{c}{\omega_p d}\right)^2\int_0^{\infty}\frac{\mathrm{d}\bar{p}\;\bar{p}^4}{\mathrm{e}^{\bar{p}}-1}\int_{x_0}^{\infty}\frac{\mathrm{d}x}{x}\;\frac{\mathrm{d}}{\mathrm{d}\omega}\left(-\frac{1}{\mathrm{e}^x-1}\right)\\
&= C\frac{\hbar v}{d^4}\left(\frac{k_B T\tau}{\hbar}\right)\left(\frac{\omega_p d}{c}\right)^2\end{aligned}$$

where

$$C = \frac{5}{32\pi^2}\int_0^{\infty}\frac{\mathrm{d}\bar{p}\;\bar{p}^4}{\mathrm{e}^{\bar{p}}-1} = 0.394$$

$$x_0 = \frac{\hbar\omega_0}{k_B T} \approx \frac{\hbar}{k_B T\tau}\left(\frac{c}{\omega_p d}\right)^2 \ll 1.$$

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